SSP Exercise 4 Solutions

1. Fermi Dirac distribution, $f(\varepsilon) = \dfrac{1}{e^{(\varepsilon-\varepsilon_F)/k_B T}+1}$

Assume conduction band occupancy <<1, $f(\varepsilon) \approx \dfrac{1}{e^{(\varepsilon-\varepsilon_F)/k_B T}} = e^{-\left(\frac{\varepsilon-\varepsilon_F}{k_B T}\right)}$

Probability of hole in valence band, 1- $f(\varepsilon) \approx 1 - \dfrac{1}{e^{(\varepsilon-\varepsilon_F)/k_B T}} = e^{-\left(\frac{\varepsilon_F-\varepsilon}{k_B T}\right)}$

Density of electrons in conduction band,

$$n = \int_{\varepsilon_c}^{\infty} f(\varepsilon) \times D_{\mathrm{c}} d\varepsilon$$

$$n = \int_{\varepsilon_c}^{\infty} f(e) D_{\mathrm{c}} d\varepsilon \approx C m*_{\mathrm{c}}^{\frac{3}{2}} \int_{\varepsilon_c}^{\infty} (\varepsilon - \varepsilon_{\mathrm{c}})^{\frac{1}{2}} \times e^{-\frac{(\varepsilon-\varepsilon_F)}{k_{\mathrm{B}} T}} d\varepsilon$$

$$n = 2\frac{(2\pi m*_{\mathrm{c}} k_{\mathrm{B}} T)^{\frac{3}{2}}}{h^3} \times e^{-\frac{(\varepsilon_{\mathrm{c}}-\varepsilon_F)}{k_{\mathrm{B}} T}} = N_{\mathrm{c}} \times e^{-\frac{(\varepsilon_{\mathrm{c}}-\varepsilon_F)}{k_{\mathrm{B}} T}}$$ **[5]**

Similarly, density of holes in valence band,

$$p = \int_{-\infty}^{\varepsilon_{\mathrm{v}}} f(\varepsilon) D_{\mathrm{v}} d\varepsilon$$

$$p = \int_{-\infty}^{\varepsilon_{\mathrm{v}}} e^{-\left(\frac{\varepsilon_F-\varepsilon}{k_B T}\right)} D_{\mathrm{v}} d\varepsilon$$

$$p = \int_{-\infty}^{\varepsilon_{\mathrm{v}}} e^{-\left(\frac{\varepsilon_F-\varepsilon}{k_B T}\right)} D_{\mathrm{v}} d\varepsilon \approx C m*_{\mathrm{v}}^{\frac{3}{2}} \int_{-\infty}^{\varepsilon_{\mathrm{v}}} (\varepsilon_v - \varepsilon)^{\frac{1}{2}} \times e^{-\frac{(\varepsilon_F-\varepsilon)}{k_{\mathrm{B}} T}} d\varepsilon$$

$$p = 2\frac{(2\pi m*_{\mathrm{v}} k_{\mathrm{B}} T)^{\frac{3}{2}}}{h^3} \times e^{-\frac{(\varepsilon_F-\varepsilon_{\mathrm{v}})}{k_{\mathrm{B}} T}} = N_{\mathrm{v}} e^{-\frac{(\varepsilon_F-\varepsilon_{\mathrm{v}})}{k_{\mathrm{B}} T}}$$ **[5]**

2. $$n_\mathrm{i} = n = p = W^{\frac{1}{2}} T^{\frac{3}{2}} e^{-\frac{\varepsilon_\mathrm{g}}{2k_\mathrm{B}T}} \quad \ldots \text{①}$$

Can use free electron concepts for electrons in the conduction band.

$\Rightarrow$ Electrical conductivity , $\sigma = \dfrac{n_i e^2 \tau}{m_\mathrm{c}^*} \propto n_i$

$\Rightarrow$ Need to find the temperature, T, for which $n_\mathrm{i} = 1.1\, n_\mathrm{i}(300\ \mathrm{K})$

Using equation ① and considering the exponential temperature terms only:

$$\frac{n_\mathrm{i}(300K)}{1.1 n_\mathrm{i}(300K)} = \frac{e^{-\frac{\varepsilon_\mathrm{g}}{2k_\mathrm{B}300}}}{e^{-\frac{\varepsilon_\mathrm{g}}{2k_\mathrm{B}T}}}$$

ε_g = 1.1 eV

$\Rightarrow$ T=301 K **[5]**